Relativistic $O(q^4)$ two-pion exchange nucleon-nucleon potential: parametrized version

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(February 9, 2008)

The chiral two-pion exchange nucleon-nucleon interaction has nowadays a rather firm conceptual basis, but depends on low-energy constants, which may be extracted from fits to data. In order to facilitate this kind of application, we present here a parametrized version of our relativistic expansion of this component of the force to $O(q^4)$, performed recently.

I. INTRODUCTION

Nuclear interactions are strongly dominated by the quarks u and d and can be accomodated into a two-flavor version of QCD. The masses of these quarks are small in the GeV scale and one is not far from the massless lagrangian limit, in which QCD is invariant under chiral $SU(2) \times SU(2)$ transformations. For this reason, low energy hadronic processes can be reliably described by means of effective lagrangians that are symmetric under the Poincaré and isospin groups and incorporate approximate chiral symmetry, realized in the Nambu-Goldstone mode.

The one-pion exchange NN potential ($OPEP$) is simple, has been well established long ago, and dominates completely partial waves with orbital momentum $L \geq 5$. The two-pion exchange potential ($TPEP$), on the other hand, is rahter complex and has become free of important umbiguities only in the 1990s, after the systematic use of chiral symmetry in its theoretical description [1–11].

Chiral perturbation theory is based on the existence of a characteristic scale q, set by both pion four-momenta and nucleon three-momenta, such that $q < 1$ GeV. Due to this technique, nowadays one understands rather well the internal hierarchies of the NN potential in terms of chiral layers. Leading terms of the chiral $TPEP$ are of order $O(q^2)$ and expansions which go up to $O(q^4)$ are already available. One of them was produced recently by our group [10,11]. We departed from a relativistic lagrangian and evaluated the relevant Feynman diagrams covariantly, without resorting to heavy baryon approximations. The so obtained T-matrix was then transformed into a potential, expressed in terms of covariant loop integrals and observable parameters. Without loss of generality, one may choose these parameters to be either the subthreshold coefficients extracted from πN scattering [12] or the low-energy contants ($LECs$) present in the effective lagrangian. These two possibilities are formally equivalent, but suitded to slightly different physical purposes. The former choice yields a closed prediction for the potential, whereas the latter gives rise to an open theoretical structure which may be used to fit NN data.

Nowadays, a rather pressing issue is to determine the extent that nature backs this picture. The comparison of chiral predictions with empirical phase shifts is not straightforward, since existing theoretical potentials are not reliable for distances smaller than 1 fm [11]. Three complementary possibilities are available for overcoming this problem. The most direct one is to use peripheral waves and rely on those windows in angular momenta and energies for which the centrifugal barrier is effective in suppressing the interaction at short distances. When this happens, the Born approximation can be used and one does not need to know the potential close to the origin. However, this kind of test is not very stringent, since peripheral waves are small and uncertainties are large

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[4,5,13]. Inclusion of more important waves requires the use of dynamical equations with regularized potentials [8]. This regularization brings necessarily extra parameters into the problem which are not constrained by chiral symmetry. One is then faced with the problem of disentangling from ones results those windows which do indeed test the symmetry. The third alternative, already employed by the Nijmegen group [7], is to assume that the theoretical potential determines correctly the interaction in a spatial window ranging from a radius R onwards and then use it as an input in phase shift analises. This procedure has already proved to be useful in the case of the $OPEP$, in the elastic regime and for waves with $L \geq 5$. Its extension to the $TPEP$ becomes not trivial due to the fact that the range of reliability of the theoretical potential depends on the chiral order one is working at.

The research on the $TPEP$ performed in the last decade has set its conceptual foundations on a rather solid basis, comparable to that of the OPEP in the late sixties. On the other hand, the $TPEP$ depends on several $LECs$, which must be extracted from eihter πN scattering data or direct fits of NN phase shifts. In general, this last kind of procedure tends to be computationally heavy, for theoretical results are usually given as cumbersome expressions. In order to make applications easier, in this work we present a parametrized version of our $O(q^4)$ relativistic configuration space $TPEP$, which is numerically accurate for distances larger than 1 fm. It is based on the theoretical expressions derived in Ref. [11] and reproduced in appendix A, as functions of the LECs.

II. PARAMETRIZED POTENTIAL

The configuration space potential has the isospin structure

$$V(\boldsymbol{r}) = V^+(\boldsymbol{r}) + \boldsymbol{\tau}^{(1)}\cdot\boldsymbol{\tau}^{(2)}\; V^-(\boldsymbol{r})\;, \tag{2.1}$$

with

$$V^{\pm}(\boldsymbol{r}) = V_C^{\pm} + V_{LS}^{\pm}\,\Omega_{LS} + V_T^{\pm}\,\Omega_T + V_{SS}^{\pm}\,\Omega_{SS} + V_Q^{\pm}\,\Omega_Q\;, \tag{2.2}$$

and $\Omega_{LS} = \boldsymbol{L}\cdot(\boldsymbol{\sigma}^{(1)} + \boldsymbol{\sigma}^{(2)})/2$, $\Omega_T = 3\,\boldsymbol{\sigma}^{(1)}\cdot\hat{\boldsymbol{r}}\;\boldsymbol{\sigma}^{(2)}\cdot\hat{\boldsymbol{r}} - \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$, $\Omega_{SS} = \boldsymbol{\sigma}^{(1)}\cdot\boldsymbol{\sigma}^{(2)}$. The form of the operator Ω_Q in configuration space is highly non-local and can be found in Ref. [14].

In Refs. [10,11] we have presented a $O(q^4)$ relativistic expansion of the $TPEP$, which is reproduced, in an alternative form, in appendix A. The configuration space potential is written in terms of numerical coefficients which multiply dimensionless functions arising form the Fourier transforms of Feynman loop integrals. The former are combinations of external parameters representing the pion and nucleon masses, μ and m, respectively, the pion decay constant f_π, the axial coupling constant g_A, and the LECs c_i and d_i. The latter are denoted by Z_i and depend on just μ, m, f_π, and $x \equiv \mu\, r$. We keep the external quantities as free and parametrize the function $Z_i \equiv (F_i, G_i)$ as

$$Z_i = -\frac{\mu}{(4\pi)^{5/2}}\left(\frac{\mu}{f_\pi}\right)^4 \left[\sum \gamma_i^n\, x^n\right] \frac{e^{-2x}}{x^2}\;. \tag{2.3}$$

The coefficients γ_i^n corresponding to the various cases are given in the tables at the end of this section. This parametrization is more than 1% accurate in the range 0.8 fm $\leq r \leq$ 10 fm.

Using the definition $\alpha \equiv \mu/m$, the profile functions are written as

$$\bullet\; V_C^+ = \frac{3g_A^4}{16}\,G_1 - \frac{3g_A^2\alpha}{2}\left\{4m\,c_1\left[2\,I_2 - I_1 - 2\alpha(H_1 - H_2)\right]\right.$$

$$\left. + \frac{m\,c_2}{3}\,\alpha\left(3\,H_1 - 2\,H_3\right) - 2m\,c_3\left[I_1 - I_3 + \alpha\Big(2\,H_1 - 2\,H_2 - H_3\Big)\right]\right\}$$

$$+ \frac{3\alpha^2}{2}\left[(4m\,c_1)^2\,H_1 + \frac{1}{5}\,(m\,c_2)^2\left(4\,H_2 - H_3\right) + (2m\,c_3)^2\left(H_1 - H_3\right)\right.$$

$$\left. - \frac{16}{3}\,m^2 c_1\,c_2\,H_2 - 16m^2 c_1\,c_3\left(2\,H_2 - H_1\right) + \frac{4}{3}\,m^2 c_2\,c_3\left(2\,H_2 - H_3\right)\right] + \frac{3g_A^6\mu^2}{16\pi\,f_\pi^2}\left(I_1 - I_3\right)$$

$$- \frac{3g_A^4\mu^2}{256\pi^2 f_\pi^2}\left\{\left[8\left(I_6 - I_8\right) - 7\left(I_5 - I_7\right)\right] + 4\pi\left[4\,I_3 + 6\,I_2 - 7\,I_1\right]\right\}, \tag{2.4}$$

$$\bullet\, V_{LS}^{+} = \frac{3g_A^4\,\alpha}{8}\,G_2 - 4g_A^2\,\alpha^2\;m\,c_2\;H_5\;, \tag{2.5}$$

$$\bullet\, V_T^{+} = -\frac{g_A^4}{16}\,G_3 + \frac{g_A^2\,\alpha^2}{3}\,(m^2\,\tilde{d}_{14} - m^2\,\tilde{d}_{15})\Big(H_3 - 3\,H_5\Big) - \frac{g_A^6\,\mu^2}{96\pi^2 f_\pi^2}\;\Big(H_3 - 3\,H_5\Big)\;, \tag{2.6}$$

$$\bullet\, V_{SS}^{+} = \frac{g_A^4}{8}\,G_4 - \frac{2g_A^2\,\alpha^2}{3}\,(m^2\,\tilde{d}_{14} - m^2\,\tilde{d}_{15})\,H_3 + \frac{g_A^6\,\mu^2}{48\pi^2 f_\pi^2}\;H_3\;, \tag{2.7}$$

$$\begin{aligned}
\bullet\, V_C^{-} &= \frac{g_A^4}{8}\,G_5 + \frac{g_A^2}{12}\Bigg\{\Bigg[2\Big(5\,H_2 - 3\,H_1\Big) - 3\,\alpha\Big(I_1 - I_3\Big) - 3\,\alpha^2\Big(2\,H1 - 2\,H2 - H_3\Big)\Bigg] \\
&+\alpha^2\Bigg[2m\,c_4\Big(5\,H_3 - 12\,H_1 + 12\,H_2\Big) - 8(m^2 d_1 + m^2 d_2)\Big(5\,H_3 + 2\,H_2 - 6\,H_1\Big) \\
&-\frac{4m^2 d_3}{5}\Big(7\,H_3 - 8\,H_2\Big) + 32m^2 d_5\Big(5\,H_2 - 3\,H_1\Big)\Bigg]\Bigg\} \\
&+\frac{1}{12}\Bigg\{H_2 + \alpha^2\Bigg[2m\,c_4\,H_3 + 8(m^2 d_1 + m^2 d_2)\Big(2\,H_2 - H_3\Big) + \frac{12m^2 d_3}{5}\Big(4\,H_2 - H_3\Big) \\
&+32m^2 d_5\,H_2\Bigg]\Bigg\} - \frac{g_A^6\,\mu^2}{288\pi^2 f_\pi^2}\;\frac{1}{50}\,(201\,H_3 + 156\,H_2 - 300\,H_1) \\
&-\frac{g_A^4\,\mu^2}{288\pi^2 f_\pi^2}\;\frac{1}{50}\Big[1250\,H_8 - 1500\,H_7 + 450\,H_6 - 346\,H_3 + 1084\,H_2 - 300\,H_1\Big] \\
&-\frac{g_A^2\,\mu^2}{144\pi^2 f_\pi^2}\Bigg\{5\,H_8 - 3\,H_7 - \frac{61}{20}\,H_3 + \frac{61}{5}\,H_2 - 3\,H_1\Bigg\} \\
&-\frac{\mu^2}{288\pi^2 f_\pi^2}\Bigg[H_8 - \frac{1}{10}\Big(14\,H_3 - 76\,H_2\Big)\Bigg]\;,
\end{aligned} \tag{2.8}$$

$$\begin{aligned}
\bullet\, V_{LS}^{-} &= \frac{g_A^4\,\alpha}{16}\,G_6 - \frac{g_A^2\,\alpha}{12}\Bigg[6\,I_4 + \alpha\,(3 - 40m\,c_4)\,H_5 + \alpha\,24m\,c_4\,H_4\Bigg] \\
&+\frac{\alpha^2}{24}\;(3 + 16m\,c_4)\,H_5\;,
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
\bullet\, V_T^{-} &= \frac{g_A^4\,\alpha}{48}\,G_7 + \frac{g_A^2\,\alpha}{144}\,(1 + 4m\,c_4)\Bigg\{6\Big(I_3 - 3\,I_4\Big) + \alpha\Bigg[\Big(8\,H_3 - 12\,H_1 + 12\,H_2\Big) \\
&-3\Big(8\,H_5 - 3\,H_4\Big)\Bigg]\Bigg\} - \frac{\alpha^2}{144}\,(1 + 4m\,c_4)^2\Big(H_3 - 3\,H_5\Big) - \frac{g_A^6\,\mu^2}{96\pi\,f_\pi^2}\Big(I_3 - 3\,I_4\Big) \\
&+\frac{g_A^4\,\mu^2}{384\pi^2 f_\pi^2}\Bigg[\Big(I_8 - 3\,I_9\Big) - 2\pi\Big(I_3 - 3\,I_4\Big)\Bigg]\;,
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
\bullet\, V_{SS}^{-} &= -\frac{g_A^4\,\alpha}{24}\,G_8 - \frac{g_A^2\,\alpha}{72}\,(1 + 4m\,c_4)\Bigg[6\,I_3 + \alpha\Big(8\,H_3 - 12\,H_1 + 12\,H_2\Big)\Bigg] \\
&+\frac{\alpha^2}{72}\,(1 + 4m\,c_4)^2\,H_3 + \frac{g_A^6\,\mu^2}{48\pi\,f_\pi^2}\,I_3 - \frac{g_A^4\,\mu^2}{192\pi^2 f_\pi^2}\Big(I_8 - 2\pi\,I_3\Big)\;.
\end{aligned} \tag{2.11}$$

γ_i^n	$-1/2$	$-3/2$	$-5/2$	$-7/2$	$-9/2$	$-11/2$
H_1	-1	-3/16	15/512	-105/8192	0.0069211	-0.002031054
H_2	-	3/2	45/32	315/1024	-0.050879	0.0105639
H_3	-	6	165/8	8715/256	27.45483	5.43256
H_4	-	2	23/8	153/256	-0.0723934	-
H_5	-	-	-3	-129/16	-3555/512	-1.33605
H_6	-3.89861	4.23305	-0.833136	-	-	-
H_7	-	5.78893	-7.63019	-2.69576	-	-
H_8	-	-	-14.3654	14.6375	39.3909	18.8729

γ_i^n	1	0	-1	-2	-3	-4	-5	-6
G_1	-	2.83823	-7.200711	38.9637	-55.5164	47.2443	-16.2395	-
G_2	-	-	-6.12315	-28.1422	-30.2813	0.023458	-15.8996	7.18869
G_3	-	0.5579	17.1039	16.8038	9.94755	3.40171	-2.7544	-
G_4	-	0.569624	15.9429	-4.26031	15.6445	-5.06641	-	-
G_5	-0.217221	-9.98415	-4.662	-36.9761	13.4087	-6.21047	-	-
G_6	-	-	7.90985	55.9568	86.3242	66.9540	-29.5680	11.8985
G_7	-	1.69219	25.5612	6.53589	160.459	-169.567	120.612	-36.7881
G_8	-	1.7661	21.2122	-9.87710	116.454	-144.344	103.063	-30.9265

γ_i^n	2	1	0	-1	-2	-3	-4
I_1	0.000483761	-0.0226386	1.53346	0.0595627	-0.0913580	0.0291743	-
I_2	-	-0.000381934	0.0158372	-1.63009	-0.660019	0.0532419	-
I_3	-	-	0.242214	-8.87827	-6.47733	-30.5206	-
I_4	-	-	-	-0.00147946	2.99191	6.86185	1.82098

γ_i^n	$3/2$	$1/2$	$-1/2$	$-3/2$	$-5/2$	$-7/2$	$-9/2$
I_5	0.168731	-6.00262	-2.18325	1.79108	-	-	-
I_6	-	-0.129344	5.46315	-2.54740	0.240122	-	-
I_7	-	-0.464737	20.2994	31.4762	-26.1396	-	-
I_8	-	-	-	-28.6452	-101.827	28.3771	99.2609
I_9	-	-	-	-0.454235	18.7535	24.4758	-31.2207

The parametrized profile functions given above depend explicitly on four well known quantities, namely m, μ, g_A, f_π, and on the less known LECs c_i and d_i. Therefore, the latter may be extracted from fits to data. When doing this, however, one has to bear in mind that, as discused in Ref. [11], the influence of the LECs over the profile functions is rather uneven. Indeed, their influence over V_C^+, V_{LS}^-, V_T^-, and V_{SS}^- is rahter strong, but barely perceptible in V_C^-, V_{LS}^+, V_T^+ and V_{SS}^+.

ACKNOWLEDGMENTS

The work of R. H. was supported by DOE Contract No.DE-AC05-84ER40150 under which SURA operates the Thomas Jefferson National Accelerator Facility, and M. R. R. and C. A. dR. were supported by FAPESP (Fundação de Amparo à Pesquisa do Estado de São Paulo).

APPENDIX A: THEORETICAL POTENTIAL

The $O(q^4)$ relativistic expansion of the TPEP produced in Refs. [10,11] was based on the evaluation of three familes of diagrams[1]. The first of them involves only pion and nucleon degrees of freedom into single loops and corresponds to the minimal realization of chiral symmetry [3]. It includes the subtraction of the iterated OPEP and yields the terms in the profile functions given below which are proportional to just g_A^4/f_π^4, g_A^2/f_π^4 or $1/f_\pi^4$. Terms proportional to $1/f_\pi^6$, on the other hand, come from two-loop processes, either in the form of t-channel contributions from the second family or s and u-channel terms embodied in the subthreshold coefficients of the third family. Finally, the third group of diagrams includes chiral corrections associated with other degrees of freedom, hidden within the LECs c_i and d_i, and gives rise to contributions which are proportional to either $(LEC)/f_\pi^4$ or $(LEC)^2/f_\pi^4$. In Ref. [11] we have expressed this last class of results in terms of the πN subthreshold coefficients. As these can be easily translated into LECs, in the present work we write the potential in terms of these constants, which appear directly into the effective lagrangians. The following expressions correspond to the updated version as described in Ref. [13].

The radial components of the potential are expressed in terms of the following profile functions[2]

$$V_C^\pm(r) = \tau^\pm\, U_C^\pm(x)\,, \tag{A1}$$

$$V_{LS}^\pm(r) = \tau^\pm\, \frac{\mu^2}{m^2}\,\frac{1}{x}\,\frac{d}{dx}\, U_{LS}^\pm(x)\,, \tag{A2}$$

$$V_T^\pm(r) = \tau^\pm\, \frac{\mu^2}{m^2}\left[\frac{d^2}{dx^2} - \frac{1}{x}\,\frac{d}{dx}\right] U_T^\pm(x)\,, \tag{A3}$$

$$V_{SS}^\pm(r) = -\tau^\pm\, \frac{\mu^2}{m^2}\left[\frac{d^2}{dx^2} + \frac{2}{x}\,\frac{d}{dx}\right] U_{SS}^\pm(x)\,, \tag{A4}$$

where $\tau^+ = 3$ and $\tau^- = 2$.

Defining $\hat{t}$ as the Laplacian operator acting on the variable $x = \mu r$ we write our profile functions as

$$-U_C^+ = \frac{3g_A^4\mu^5}{256\pi^2 f_\pi^4}\left\{\left[1 - \left(1-\alpha^2/4\right)\hat{t} + \hat{t}^2/4\right]\, S_\times - \left[1 - \left(1+\alpha^2/4\right)\hat{t} + \hat{t}^2/4\right]\, S_b\right.$$

$$\left. -\alpha\,(1-\hat{t}/2)\left[2S_a + \hat{t}\,S_t\right] + \alpha^2\,\hat{t}^2\,S_\ell\right\} - \frac{3g_A^2\mu^5\,\alpha}{32\pi^2 f_\pi^4}\left\{4m\,c_1\left[\left[2(1-\hat{t}/4) - 1\right] S_t - \frac{\alpha\,\hat{t}}{2}\,S_\ell\right]\right.$$

$$\left. +\frac{m\,c_2}{3}\,\alpha\left[3 - 2(1-\hat{t}/4)\,\hat{t}\right] S_\ell - 2m\,c_3\left[\left[1 - (1-\hat{t}/4)\,\hat{t}\right] S_t + \alpha\left[\hat{t}/2 - (1-\hat{t}/4)\,\hat{t}\right] S_\ell\right]\right\}$$

$$+\frac{3\mu^5\,\alpha^2}{32\pi^2 f_\pi^4}\left\{(4m\,c_1)^2\, S_\ell + \frac{1}{5}\,(m\,c_2)^2\left[4(1-\hat{t}/4) - (1-\hat{t}/4)\,\hat{t}\right] S_\ell\right.$$

$$+(2m\,c_3)^2\,\left[1 - (1-\hat{t}/4)\,\hat{t}\right] S_\ell - \frac{16}{3}\,m^2 c_1\,c_2\,(1-\hat{t}/4)\,S_\ell - 16m^2 c_1\,c_3\,\left[2(1-\hat{t}/4) - 1\right] S_\ell$$

[1] Please see section II of Ref. [11], for a detailed discussion of the meaning and dynamical contents of these families of diagrams, which are given in its Fig.2.

[2] Please see Eqs. (3.4)-(3.8) of Ref. [11]. Note that in Eq. (3.8) a multiplication factor of μ^3 is missing.

$$+\frac{4}{3}\,m^2 c_2\,c_3\,\Big[2(1-\hat{t}/4)-(1-\hat{t}/4)\,\hat{t}\Big]\,S_\ell\Bigg\}+\frac{3g_A^6\mu^7}{256\pi^3 f_\pi^6}\,\Big[1-(1-\hat{t}/4)\,\hat{t}\Big]\,S_t$$

$$-\frac{3g_A^4\mu^7}{4096\pi^4 f_\pi^6}\Bigg\{\Bigg\{8(1-\hat{t}/4)-7-\Big[8(1-\hat{t}/4)^2-7(1-\hat{t}/4)\Big]\,\hat{t}\Bigg\}\,S_{tt}$$

$$+4\pi\,\Big[4(1-\hat{t}/4)\,\hat{t}+6(1-\hat{t}/4)-7\Big]\,S_t\Bigg\}\;, \tag{A5}$$

$$-U_{LS}^{+}=\frac{3m\,g_A^4\mu^4}{128\pi^2 f_\pi^4}\Bigg\{(1-\hat{t}/2)\,(\tilde{S}_b-S_t)-(3/2-5\,\hat{t}/8)\,S_a+\frac{\alpha}{4}\,(1{+}2\hat{t}{-}\hat{t}^2/2)\,(S_\times{+}S_b)$$

$$+2\alpha\,\hat{t}\,S_\ell\Bigg\}-\frac{g_A^2\mu^5}{4\pi^2 f_\pi^4}\;m\,c_2\;(1{-}\hat{t}/4)\,S_\ell\;, \tag{A6}$$

$$-U_T^{+}=-U_{SS}^{+}/2=-\frac{m^2 g_A^4\mu^3}{256\pi^2 f_\pi^4}\Bigg\{(1-\hat{t}/4)\,S_b+\frac{\alpha}{2}\,\Big[(1-\hat{t}/2)\,(S_t-\tilde{S}_b)+(1-\hat{t}/4)\,S_a\Big]$$

$$+\Big[\big(1-\alpha^2/4\big)-\big(1-\alpha^2\big)\,\hat{t}/4-\alpha^2\,\hat{t}^2/16\Big]\,S_\times\Bigg\}+\frac{g_A^2\mu^5}{48\pi^2 f_\pi^4}\,(m^2\,\tilde{d}_{14}-m^2\,\tilde{d}_{15})\,(1-\hat{t}/4)\,S_\ell$$

$$-\frac{m^2\,g_A^6\mu^5}{1536\pi^4 f_\pi^6}\,(1-\hat{t}/4)\,S_\ell\;, \tag{A7}$$

$$-U_C^{-}=\frac{g_A^4\mu^5}{128\pi^2 f_\pi^4}\Bigg\{\Big[1-\big(1-\alpha^2/4\big)\,\hat{t}+\big(1-\alpha^2\big)\,\hat{t}^2/4+\alpha^2\,\hat{t}^3/16\Big]\,S_\times$$

$$+\Big[1-\big(1+\alpha^2/4\big)\,\hat{t}+\big(1+\alpha^2\big)\,\hat{t}^2/4-\alpha^2\,\hat{t}^3/16\Big]\,S_b$$

$$+\alpha\Big[(2-3\hat{t}+\hat{t}^2)\,S_t+(2-\hat{t})S_a\Big]-\Big[10/3-\big(11/6-\alpha^2\big)\,\hat{t}-\alpha^2\,\hat{t}^2/2\Big]\,S_\ell\Bigg\}$$

$$+\frac{g_A^2\mu^5}{192\pi^2 f_\pi^4}\,\Bigg\{2\,\Big[5(1-\hat{t}/4)-3\Big]\;S_\ell-3\,\alpha\,\Big[1-(1-\hat{t}/4)\,\hat{t}\Big]\,S_t-3\,\alpha^2\,\Big[\hat{t}/2-(1-\hat{t}/4)\,\hat{t}\Big]\,S_\ell$$

$$+\alpha^2\,\Bigg[2m\,c_4\;\Big[5(1-\hat{t}/4)\,\hat{t}-3\,\hat{t}\Big]\;S_\ell-8(m^2 d_1+m^2 d_2)\;\Big[5(1-\hat{t}/4)\,\hat{t}+2(1-\hat{t}/4)-6\Big]\;S_\ell$$

$$-\frac{4m^2 d_3}{5}\;\Big[7(1-\hat{t}/4)\,\hat{t}-8(1-\hat{t}/4)\Big]\;S_\ell+32m^2 d_5\;\Big[5(1-\hat{t}/4)-3\Big]\;S_\ell\Bigg]\Bigg\}$$

$$+\frac{\mu^5}{192\pi^2 f_\pi^4}\Bigg\{(1-\hat{t}/4)\,S_\ell+\alpha^2\Bigg[2m\,c_4\,\Big[(1-\hat{t}/4)\,\hat{t}\Big]\,S_\ell$$

$$+8(m^2 d_1+m^2 d_2)\,\Big[2(1-\hat{t}/4)-(1-\hat{t}/4)\,\hat{t}\Big]\,S_\ell+\frac{12m^2 d_3}{5}\,\Big[4(1-\hat{t}/4)-(1-\hat{t}/4)\,\hat{t}\Big]\,S_\ell$$

$$+32m^2 d_5\,(1-\hat{t}/4)\,S_\ell\Bigg]\Bigg\}-\frac{g_A^6\mu^7}{4608\pi^4 f_\pi^6}\;\frac{1}{50}\,\Big[201(1-\hat{t}/4)\,\hat{t}+156(1-\hat{t}/4)-300\Big]\;S_\ell$$

$$-\frac{g_A^4\mu^7}{4608\pi^4 f_\pi^6}\;\frac{1}{50}\Bigg\{\Big[1250(1-\hat{t}/4)^2-1500(1-\hat{t}/4)+450\Big]\,S_{\ell\ell}$$

$$+\Big[-346(1-\hat{t}/4)\,\hat{t}+1084(1-\hat{t}/4)-300\Big]\;S_\ell\;\Bigg\}$$

$$-\frac{g_A^2\mu^7}{2304\pi^4 f_\pi^6}\left\{\left[5(1-\hat{t}/4)^2-3(1-\hat{t}/4)\right]\,S_{\ell\ell}+\left[-\frac{61}{20}\,(1-\hat{t}/4)\,\hat{t}+\frac{61}{5}\,(1-\hat{t}/4)-3\right]\,S_\ell\right\}$$

$$-\frac{\mu^7}{4608\pi^4 f_\pi^6}\left\{(1-\hat{t}/4)^2\,S_{\ell\ell}-\frac{1}{10}\left[14(1-\hat{t}/4)\,\hat{t}-76(1-\hat{t}/4)\right]\,S_\ell\right\}, \tag{A8}$$

$$-U_{LS}^- = \frac{m\,g_A^4\mu^4}{256\pi^2 f_\pi^4}\left\{(6-5\,\hat{t}/2)\,S_a-(4-2\,\hat{t})\,\tilde{S}_b+4\,S_t\right.$$

$$\left.+\alpha\left[2\,(1-\hat{t}/4)\,S_\ell+(1-\hat{t}/2)^2\,(S_\times-S_b)\right]\right\}$$

$$-\frac{m\,g_A^2\mu^4}{192\pi^2 f_\pi^4}\left[6\,(1-\hat{t}/4)\,S_t+\alpha\,(3-40m\,c_4)\,(1-\hat{t}/4)\,S_\ell+\alpha\,24m\,c_4\,S_\ell\right]$$

$$+\frac{\mu^5}{384\pi^2 f_\pi^4}\,(3+16m\,c_4)\,(1-\hat{t}/4)\,S_\ell\;, \tag{A9}$$

$$-U_T^- = -U_{SS}^-/2 = \frac{m\,g_A^4\mu^4}{768\pi^2 f_\pi^4}\left[(1-\hat{t}/2)\,\tilde{S}_b+(1-\hat{t}/4)\,(S_a-2\,S_t)-\frac{\alpha}{12}\,(16-7\,\hat{t})\,S_\ell\right]$$

$$+\frac{m\,g_A^2\mu^4}{2304\pi^2 f_\pi^4}\,(1+4m\,c_4)\left\{6\,(1-\hat{t}/4)\,S_t+\alpha\left[8\,(1-\hat{t}/4)-3\right]\,S_\ell\right\}$$

$$-\frac{\mu^5}{2304\pi^2 f_\pi^4}\,(1+4m\,c_4)^2\,(1-\hat{t}/4)\,S_\ell-\frac{m^2 g_A^6\mu^5}{1536\pi^3 f_\pi^6}\,(1-\hat{t}/4)\,S_t$$

$$+\frac{m^2 g_A^4\mu^5}{6144\pi^4 f_\pi^6}\left[(1-\hat{t}/4)^2\,S_{tt}-2\pi\,(1-\hat{t}/4)\,S_t\right]. \tag{A10}$$

The dimensionless functions $S_i(x)$ carry the spatial dependence of the potential and are given by Eqs. (3.16)-(3.23) of Ref. [11].

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